

**Electron Electric Dipole Moment
in Mirror Fermion Model
with Electroweak Scale Non-sterile
Right-handed Neutrinos**

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Abstract

The electric dipole moment of the electron is studied in detail in an extended mirror fermion model with the following unique features of (a) the right-handed neutrinos are non-sterile and have masses at the electroweak scale, and (b) equipping a horizontal symmetry of the tetrahedral group in the lepton and scalar sectors. We analyze the parameter space of the model by using the latest ACME experimental limit on the electron electric dipole moment. Other low energy experimental observables such as the anomalous magnetic dipole moment of the muon, charged lepton flavor violating processes like muon decays into electron plus photon and muon-to-electron conversion in titanium, gold and lead are also considered in our analysis for comparison. In addition to the well-known CP violating Dirac phase in the neutrino mixing matrix, the dependence of additional phases of the new Yukawa couplings in the model is studied in detail for all these observables.

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I. INTRODUCTION

The particle spectrum of the Standard Model (SM) has now been completed by the discovery of the 125 GeV Higgs boson at the Large Hadron Collider (LHC). Nevertheless, many questions remained unanswered within the SM. On the conceptual side, we do not understand the instability of the Higgs boson mass under quantum corrections, indicating that SM is very sensitive to new physics beyond the TeV scale; while on the phenomenological side, we have issues like the Baryon Asymmetry of the Universe (BAU), dark matter, and neutrino masses etc. The current popular view is that SM is just a low energy effective theory of a better one at a higher scale with new physics that can address some or all of the above issues. Indeed many beautiful ideas had been suggested in the literature to solve some of these issues. Current LHC constraint is already quite stringent on the scale of new physics $\Lambda_{\text{NP}} \sim$ a half to a few TeV, should the new physics be supersymmetry or extra dimension or sequential fourth generation, or technicolor etc. While one should continue the direct searches for new particles at the LHC, looking for new physics indirectly from low energy observables where new particles only exist virtually at the spooky loop level is an important alternative avenue. Historically one can recall that the charm quark was predicted long before its discovery by the GIM mechanism [1], which was engaged to suppress flavor changing neutral currents in the box diagrams of the $K\bar{K}$ kaon system.

The electric dipole moment (EDM) of elementary particle, like electron or neutron, is one such low energy observable which is sensitive to new CP violating phases from new physics. As is well known the CP violation phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix is too minuscule to account for the BAU, characterized by the ratio of the net baryon number density to the entropy density in the Universe [2],

$$Y_B \equiv n_B/s = (8.61 \pm 0.09) \times 10^{-11} .$$

Moreover, the SM contribution to the electron EDM from the CKM CP violation phase must be arise from at least four-loop [3–6]. The reasons are as follows: Due to the structure of the particle exchange symmetry in the loop integrals of the various diagrams, the W boson EDM vanishes to two-loop order in CKM model, but it can be non-vanishing with one more loop dressing by the gluons. Then by attaching the two external W boson lines of the three-loop diagrams to the electron one can generate the electron EDM in SM. Thus the resulting electron EDM (d_e) in SM is a four-loop result, estimated to be $\sim 8 \times 10^{-41} e \cdot \text{cm}$ [6], which is twelve order of magnitude below the current experimental limit (see below). Therefore a positive measurement of the electron EDM at the current sensitivities of various experiments or their projected improvements in the near future would definitely imply new sources of CP violation. The new CP violating phases might then be helpful to solve the BAU puzzle.

The latest measurement of the electron EDM was done by the ACME Collaboration [7] using the polar molecule thorium monoxide (ThO) just a few years back,

$$d_e = (-2.1 \pm 3.7_{\text{stat}} \pm 2.5_{\text{syst}}) \times 10^{-29} e \cdot \text{cm} . \quad (1)$$

This corresponds to a 90% confidence limit,

$$|d_e| < 8.7 \times 10^{-29} e \cdot \text{cm} , \quad (2)$$

which is an improvement by a factor of 12 over the previous best measurements.

In this work, we study the electron EDM in a class of mirror fermion model proposed some time ago by one of the authors [8]. We will demonstrate that the above ACME limit can put stringent constraints on the parameter space of the extended mirror fermion model discussed below.

Here we briefly review the salient features of the original mirror model [8]. In contrast with the left-right symmetric models, the gauge group was chosen to be the same as SM, only mirror fermions were introduced. Right-handed neutrinos

were introduced as well, but instead of being sterile singlets, they were put inside a right-handed weak doublet with the mirror charged leptons for each generation. In addition to the SM Higgs doublet, the Georgi-Machacek triplets [9, 10] were also needed to provide Majorana masses for right-handed neutrinos. To obtain the correct electroweak symmetry breaking pattern, the triplet vacuum expectation value (VEV) should be around the electroweak scale as well. Thus the non-sterile right-handed neutrinos have masses at the electroweak scale which imply immediate consequences at the LHC! Furthermore, an electroweak scalar singlet was also brought into the model to generate tiny Dirac neutrino masses of order eV through small enough VEV and provide small mixings between SM fermions and their mirrors.

Recently, many phenomenological implications of the mirror model [8] have been explored further. At the risk of being against humbleness, we summarize what we have been done in a series of works involving various collaborations: In [11], the model was challenged by the electroweak precision measurements. It was shown that the dangerously large contributions to the oblique parameters from the mirror fermions (especially the T parameter) can be tamed by the opposite contributions from the Higgs triplets. In [12], the original mirror model was extended by adding a mirror Higgs doublet so as to accommodate the LHC data for the SM Higgs signal strengths of various channels. Searches for the mirror fermions at the LHC were studied in [13] for mirror quarks and [14] for mirror leptons. In [15], the neutrino and charged lepton masses and mixings were discussed in the mirror model with a horizontal A_4 symmetry imposed on the lepton sector. Subsequently, in [16], the charged lepton flavor violating (CLFV) radiative decay $\mu \rightarrow e\gamma$ was studied in details in this mirror model with the A_4 symmetry extension, updating an earlier calculation [17] done for the original model. Moreover, the $\mu - e$ conversion in nuclei was also studied [18]. In [19], the CLFV Higgs decay $h(125\text{ GeV}) \rightarrow \mu\tau$ was studied for the extended mirror model with a mirror Higgs doublet [12].

There exists huge amount of studies of the electron EDM in the literature for other new physics models with CP violation, for example the generic two Higgs doublet model [20], minimal supersymmetric standard model (MSSM) [21–23], models with sterile neutrinos [24], *etc.* For recent reviews on this topics, see for example [25–28]. We focus on the electron EDM in the mirror model with the A_4 symmetry as discussed in [15].

This paper is organized as follows. In Sect. II, we give some more details of the model by spelling out the relevant lepton and scalar spectra, their A_4 assignments, the new Yukawa couplings and the implied small mixings between the SM leptons and their mirror counterparts. In Sect. III, we present the formulas of the lepton EDMs. In Sect. IV, we first discuss the assumptions and scenarios used in the numerical analysis and then present the numerical results for the electron EDM. We conclude in Sect. V. Some useful formulas are relegated to the end by an Appendix.

II. BRIEF REVIEW OF THE MODEL

In this section we highlight the original mirror fermion model discussed in [8] and its recent A_4 extension [15].

A. Particle Content and Its A_4 Assignments

The particle content of leptons and bosons of the model are shown in Table I, together with their quantum number assignments under $SU(2) \times U(1)_Y \times A_4$. For each generation i , we introduce the mirror fields l_{Ri}^M and e_{Li}^M of the SM lepton doublet l_{Li} and singlet e_{Ri} respectively. For the scalars, Φ_M is the mirror Higgs doublet of Φ ; ξ and $\tilde{\chi}$ are the Georgi-Machacek triplets; and ϕ_{0S} and ϕ_{iS} ($i = 1, 2, 3$) are all singlets. The A_4 assignments of these particles are also listed in Table I.

The singlet scalars $\phi_{0S}, \vec{\phi}_S = (\phi_{1S}, \phi_{2S}, \phi_{3S})$ are the only fields connecting the

| Fields | $(SU(2), U(1)_Y ; A_4)$ |
|---|-------------------------|
| $l_{Li} = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_i$, $l_{Ri}^M = \begin{pmatrix} \nu_R \\ e_R^M \end{pmatrix}_i$ | $(2, -\frac{1}{2} ; 3)$ |
| e_{Ri} , e_{Li}^M | $(1, -1 ; 3)$ |
| ϕ_{0S} | $(1, 0 ; 1)$ |
| ϕ_{iS} | $(1, 0 ; 3)$ |
| Φ , Φ_M | $(2, \frac{1}{2} ; 1)$ |
| ξ | $(3, 0 ; 1)$ |
| $\tilde{\chi}$ | $(3, 2 ; 1)$ |

TABLE I. The electroweak quantum numbers of the lepton and scalar sectors in the extended mirror model together with their assignments under the horizontal A_4 symmetry.

SM fermions and their mirror counterparts. Recall that the tetrahedron symmetry group A_4 has four irreducible representations $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, and $\mathbf{3}$ with the following multiplication rule ¹:

$$\begin{aligned} \mathbf{3} \times \mathbf{3} = & \mathbf{3}_1(23, 31, 12) + \mathbf{3}_2(32, 13, 21) \\ & + \mathbf{1}(11 + 22 + 33) + \mathbf{1}'(11 + \omega^2 22 + \omega 33) + \mathbf{1}''(11 + \omega 22 + \omega^2 33) \end{aligned} \quad (3)$$

where $\omega = e^{2\pi i/3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. In the gauge eigenbasis (fields with superscript 0), one can write down the following A_4 invariant Yukawa couplings,

$$\begin{aligned} -\mathcal{L}_S = & g_{0S}\phi_{0S}(\bar{l}_L^0 l_R^{0M})_{\mathbf{1}} + g_{1S}\vec{\phi}_S \cdot (\bar{l}_L^0 \times l_R^{0M})_{\mathbf{3}_1} + g_{2S}\vec{\phi}_S \cdot (\bar{l}_L^0 \times l_R^{0M})_{\mathbf{3}_2} \\ & + g'_{0S}\phi_{0S}(\bar{e}_R^0 e_L^{0M})_{\mathbf{1}} + g'_{1S}\vec{\phi}_S \cdot (\bar{e}_R^0 \times e_L^{0M})_{\mathbf{3}_1} + g'_{2S}\vec{\phi}_S \cdot (\bar{e}_R^0 \times e_L^{0M})_{\mathbf{3}_2} + \text{H.c.} \end{aligned} \quad (4)$$

After the scalar singlets develop VEVs with $v_0 = \langle \phi_{0S} \rangle$ and $v_i = \langle \phi_{iS} \rangle$, one obtains

¹ $\mathbf{3}_1$ is differ from $\mathbf{3}_2$ because A_4 is nonabelian.

the neutrino mass matrix from the first line of (4) [15],

$$M_\nu^{\text{Dirac}} = \begin{pmatrix} g_{0S}v_0 & g_{1S}v_3 & g_{2S}v_2 \\ g_{2S}v_3 & g_{0S}v_0 & g_{1S}v_1 \\ g_{1S}v_2 & g_{2S}v_1 & g_{0S}v_0 \end{pmatrix}. \quad (5)$$

Moreover, mixings among SM charged leptons and their mirrors can be induced from the second line of (4). These mixing effects are proportional to the ratios of the singlet VEVs and the SM Higgs VEV. Hence they are small and will be ignored in our analysis. Hermiticity of the M_ν^{Dirac} implies $g_{2S} = g_{1S}^*$. Furthermore, if one assumes $v_i = v$, M_ν^{Dirac} reduces to

$$M_\nu^{\text{Dirac}} = \begin{pmatrix} g_{0S}v & g_{1S}v & g_{1S}^*v \\ g_{1S}^*v & g_{0S}v & g_{1S}v \\ g_{1S}v & g_{1S}^*v & g_{0S}v \end{pmatrix}, \quad (6)$$

which can be diagonalized by unitary transformation, *i.e.* $U_L^{\nu\dagger} M_\nu^D U_R^\nu = M_\nu^{\text{Diag}}$ with

$$U_L^\nu = U_R^\nu = U_\nu = U_{CW} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}. \quad (7)$$

We note that the matrix U_{CW} in (7) was first discussed long time ago by Cabibbo [29] and also by Wolfenstein [30] in the context of CP violation in three generations of neutrino oscillations.

B. Mixings

Let $U_{L,R}^l$ and $U_{R,L}^{l^M}$ be the unitary matrices relating the gauge eigenstates and the mass eigenstates (fields without superscripts 0) for the charged lepton fields defined as

$$l_L^0 = U_L^l l_L, \quad e_R^0 = U_R^l e_R, \quad l_R^{M,0} = U_R^{l^M} l_R^M, \quad e_L^{M,0} = U_L^{l^M} e_L^M. \quad (8)$$

TABLE II. Matrix elements for the four auxiliary $M^k (k = 0, 1, 2, 3)$ where $\omega \equiv \exp(i2\pi/3)$ and g_{0S} and g_{1S} are complex Yukawa couplings. M'^k can be obtained from M^k with the following substitutions $g_{0S} \rightarrow g'_{0S}$ and $g_{1S} \rightarrow g'_{1S}$.

| M_{jn}^k | Value |
|--|---|
| $M_{12}^0, M_{13}^0, M_{21}^0, M_{23}^0, M_{31}^0, M_{32}^0$ | 0 |
| $M_{11}^0, M_{22}^0, M_{33}^0$ | g_{0S} |
| $M_{11}^1, M_{11}^2, M_{11}^3; M_{23}^1, M_{32}^1$ | $\frac{2}{3}\text{Re}(g_{1S})$ |
| $M_{22}^1, M_{22}^2, M_{22}^3; M_{13}^1, M_{31}^1$ | $\frac{2}{3}\text{Re}(\omega^* g_{1S})$ |
| $M_{33}^1, M_{33}^2, M_{33}^3; M_{12}^1, M_{21}^1$ | $\frac{2}{3}\text{Re}(\omega g_{1S})$ |
| M_{12}^2, M_{21}^3 | $\frac{1}{3}(g_{1S} + \omega g_{1S}^*)$ |
| M_{12}^3, M_{21}^2 | $\frac{1}{3}(g_{1S}^* + \omega^* g_{1S})$ |
| M_{13}^2, M_{31}^3 | $\frac{1}{3}(g_{1S} + \omega^* g_{1S}^*)$ |
| M_{13}^3, M_{31}^2 | $\frac{1}{3}(g_{1S}^* + \omega g_{1S})$ |
| M_{23}^2, M_{32}^3 | $\frac{2\omega^*}{3}\text{Re}(g_{1S})$ |
| M_{23}^3, M_{32}^2 | $\frac{2\omega}{3}\text{Re}(g_{1S})$ |

In terms of the physical mass eigenstate fields the Yukawa couplings in (4) read

$$\mathcal{L}_S^l = - \sum_{k=0}^3 \sum_{i,m=1}^3 (\bar{l}_{Li} \mathcal{U}_{im}^{Lk} \ell_{Rm}^M + \bar{e}_{Ri} \mathcal{U}_{im}^{Rk} e_{Lm}^M) \phi_{kS} + \text{H.c.} \quad (9)$$

The coupling coefficients \mathcal{U}_{im}^{Lk} and \mathcal{U}_{im}^{Rk} are given by [16]

$$\begin{aligned} \mathcal{U}_{im}^{Lk} &\equiv \left(U_{\text{PMNS}}^\dagger \cdot M^k \cdot U_{\text{PMNS}}^M \right)_{im} , \\ &= \sum_{j,n=1}^3 \left(U_{\text{PMNS}}^\dagger \right)_{ij} M_{jn}^k (U_{\text{PMNS}}^M)_{nm} , \end{aligned} \quad (10)$$

$$\begin{aligned} \mathcal{U}_{im}^{Rk} &\equiv \left(U_{\text{PMNS}}'^\dagger \cdot M'^k \cdot U_{\text{PMNS}}'^M \right)_{im} , \\ &= \sum_{j,n=1}^3 \left(U_{\text{PMNS}}'^\dagger \right)_{ij} M_{jn}'^k (U_{\text{PMNS}}'^M)_{nm} , \end{aligned} \quad (11)$$

where the matrix elements for the four auxiliary matrices $M^k (k = 0, 1, 2, 3)$ are listed in Table II, and $M_{jn}^{'k}$ can be obtained from M_{jn}^k with the following substitutions for the Yukawa couplings $g_{0S} \rightarrow g'_{0S}$ and $g_{1S} \rightarrow g'_{1S}$. U_{PMNS} is the usual neutrino mixing matrix defined as

$$U_{\text{PMNS}} = U_{\nu}^{\dagger} U_L^l. \quad (12)$$

Analogously its mirror and right-handed counter-parts U_{PMNS}^M , U'_{PMNS} and U'^M_{PMNS} are defined as

$$U_{\text{PMNS}}^M = U_{\nu}^{\dagger} U_R^{l^M}, \quad (13)$$

$$U'_{\text{PMNS}} = U_{\nu}^{\dagger} U_R^l, \quad (14)$$

and

$$U'^M_{\text{PMNS}} = U_{\nu}^{\dagger} U_L^{l^M}. \quad (15)$$

Certainly among these four PMNS-type mixing matrices, only U_{PMNS} in (12) has been determined experimentally.

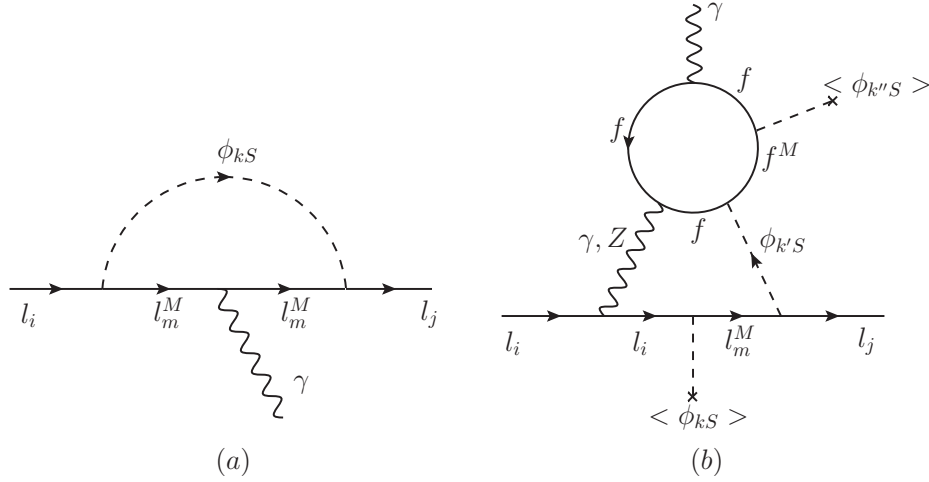


FIG. 1. Feynman diagrams contributing to charged lepton EDM in mirror fermion model. (a) one-loop diagram and (b) two-loop Barr-Zee type diagram.

III. CHARGED LEPTON ELECTRIC DIPOLE MOMENTS

The electric dipole moment (EDM) operator is defined as

$$\mathcal{L}_{l_i}^{\text{EDM}} = -i \frac{d_{l_i}}{2} \bar{l}_i \sigma^{\mu\nu} \gamma_5 l_i F_{\mu\nu} , \quad (16)$$

where $F_{\mu\nu}$ is the electromagnetic field strength and the coefficient d_{l_i} is the electric dipole moment for the i -th generation charged lepton l_i .

The one-loop and two-loop Feynman diagrams contributing to the charged lepton EDMs in the mirror model that we are discussing are depicted in Fig. 1. The two-loop Barr-Zee type diagram [31] in Fig. 1b is completely negligible due to the mixings between SM fermions and their mirrors which are proportional to the very small VEVs of the singlets. We will focus on the one-loop diagram in Fig. 1a. The one-loop amplitude for the process $l_i^-(p) \rightarrow l_j^-(p') + \gamma(q)$ has been computed in [16] with the following matrix element,

$$\mathcal{M}(l_i^- \rightarrow l_j^- \gamma) = \epsilon_\mu^*(q) \bar{u}_j(p') \{ i \sigma^{\mu\nu} q_\nu [C_L^{ij} P_L + C_R^{ij} P_R] \} u_i(p) , \quad (17)$$

where $P_{L,R} = (1 \mp \gamma_5)/2$ are the chiral projection operators and the coefficients $C_{L,R}^{ij}$ are summarized in the Appendix for convenience.

The amplitude \mathcal{M} in Eq. (17) can be reproduced by the following Fermi interaction

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2} \bar{l}_j \{ i \sigma^{\mu\nu} [C_L^{ij} P_L + C_R^{ij} P_R] \} l_i F_{\mu\nu} . \quad (18)$$

Comparing with the lepton EDM Lagrangian in Eq. (16), we can extract the electric dipole moment d_{l_i} as [16]

$$\begin{aligned} d_{l_i} &= \frac{i}{2} (C_L^{ii} - C_R^{ii}) , \\ &= +\frac{e}{16\pi^2} \text{Im} \sum_{k=0}^3 \sum_{m=1}^3 \frac{1}{m_{l_m^M}} \mathcal{U}_{im}^{Lk} (\mathcal{U}_{im}^{Rk})^* \mathcal{J} \left(\frac{m_{\phi_{kS}}^2}{m_{l_m^M}^2} \right) , \end{aligned} \quad (19)$$

where $m_{l_m^M}$ and $m_{\phi_{kS}}$ are mirror leptons and scalar Higgs masses respectively, \mathcal{U}_{im}^{Lk} and \mathcal{U}_{im}^{Rk} are defined in (10) and (11), and $\mathcal{J}(r)$ is a loop function defined in the Appendix (Eq. (37)).

IV. ANALYSIS

We will first discuss the parameter space entered in our numerical analysis. Our approach here is similar to those adopted in previous works [16, 19].

- The six Yukawa couplings $g_{0S}, g_{1S}, g_{2S}, g'_{0S}, g'_{1S}, g'_{2S}$ are in general complex. Recall that we have the following relations $g_{2S} = (g_{1S})^*$ and $g'_{2S} = (g'_{1S})^*$ due to the reality of the eigenvalues of the Dirac neutrino mass matrix. Thus we will write the couplings as follows:

$$g_{0S} = |g_{0S}| e^{i\delta_0}, g_{1S} = |g_{1S}| e^{i\delta_1}, g'_{0S} = |g'_{0S}| e^{i\delta'_0}, g'_{1S} = |g'_{1S}| e^{i\delta'_1}. \quad (20)$$

The new phases in these Yukawa couplings are the new sources of CP violation. It is useful to define the following combinations

$$\alpha = \delta_0 - \delta'_0, \quad \beta = \delta_1 - \delta'_1. \quad (21)$$

For $\delta_{0,1}$ and $\delta'_{0,1}$ range from 0 to 2π , α and β range from -2π to 2π .

- For the masses of the singlet scalars ϕ_{kS} , we assume

$$m_{\phi_{0S}} \approx m_{\phi_{kS}} = m_S = 1 \text{ GeV}. \quad (22)$$

Recall that in the seesaw mechanism the light neutrino mass is $m_\nu^{\text{light}} \sim (m_\nu^D)^2/M_R$. If the neutrino Dirac mass m_ν^D is generated at the electroweak scale, one must require the right-handed neutrino mass scale M_R to be at the grand unification scale in order to achieve light neutrino mass m_ν^{light} of order eV. However, in the electroweak scale seesaw mechanism [8], $m_\nu^D \sim g_S \langle \phi_S \rangle$ and M_R is of the order electroweak scale. To achieve eV light neutrino mass, one needs

$$g_S \langle \phi_S \rangle \sim \sqrt{M_R/\text{TeV}} \times \text{MeV}.$$

Thus for $M_R \sim v_{\text{SM}} = 246 \text{ GeV}$, as g_S varies from 10^{-5} to 1, $\langle \phi_S \rangle$ varies from 50 GeV to 0.5 MeV. The mass of the singlet $m_{\phi_S} \sim \lambda_S \langle \phi_S \rangle$ where λ_S is

a generic quartic coupling of order one in the scalar potential. So we choose the common mass m_S to be 1 GeV in (22) as a nominal value.

Similarly for the mirror lepton masses, we assume they are degenerate, *i.e.*

$$m_{l_k^M} = m_M , \quad (23)$$

and vary the common mass m_M from 100 to 800 GeV. Thus $m_M \gg m_S$, the loop function $\mathcal{J}(m_{\phi_{kS}}^2/m_{l_m^M}^2) \approx \mathcal{J}(0) = 1/2$ which is not sensitive to the masses of the singlets and the mirror leptons.

With these assumptions, the electric dipole moment in (19) can be simplified as

$$d_{l_i} \approx + \frac{e}{32\pi^2} \frac{1}{m_M} J_i , \quad (24)$$

where

$$J_{l_i} \equiv \text{Im} \sum_{k=0}^3 \sum_{m=1}^3 \mathcal{U}_{im}^{Lk} (\mathcal{U}_{im}^{Rk})^* . \quad (25)$$

- For the PMNS matrix it is most commonly parameterized by three mixing angles and one phase according to

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{23}c_{13} \end{pmatrix}$$

where $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$ with $\theta_{ij} \in [0, \pi/2]$ being the mixing angles and $\delta_{\text{CP}} \in [0, 2\pi]$ being the CP-violating Dirac phase. We have ignored the Majorana phase matrix in this analysis. We will set $\delta_{\text{CP}} = -\pi/2$ (or equivalently $3\pi/2$) as suggested by recent data of the different appearance rates for $\nu_\mu \rightarrow \nu_e$ [32, 33] and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ [32], as well as $\bar{\nu}_\mu$ disappearance rate [34] in various experiments, which is consistent with the most recent global analysis of neutrino oscillation data [35]. The current global fit results of three mixing angles were given in Table 2 of Ref. [35]. For convenience, we list them in Table III here.

TABLE III. The current global fit results ($\pm 1\sigma$) of three mixing angles taken from [35].

| Mixing angles | Normal Hierarchy | Inverted Hierarchy |
|----------------------|------------------------------|------------------------------|
| $\sin^2 \theta_{12}$ | $0.304^{+0.013}_{-0.012}$ | $0.304^{+0.013}_{-0.012}$ |
| $\sin^2 \theta_{23}$ | $0.452^{+0.052}_{-0.028}$ | $0.579^{+0.025}_{-0.037}$ |
| $\sin^2 \theta_{13}$ | $0.0218^{+0.0010}_{-0.0010}$ | $0.0219^{+0.0011}_{-0.0010}$ |

- For the three unknown PMNS matrices we assume that they are equal to each other and study the following two scenarios:

– Scenario A

$$U_{\text{PMNS}}^M = U'_{\text{PMNS}} = U'^M_{\text{PMNS}} = U_{\text{CW}}^\dagger$$

– Scenario B

$$U_{\text{PMNS}}^M = U'_{\text{PMNS}} = U'^M_{\text{PMNS}} = U_{\text{PMNS}}$$

- For comparisons, we include in our analysis the CLFV processes $\mu - e$ conversion in nuclei (titanium, gold and lead) and $\mu \rightarrow e\gamma$, and the muon anomalous magnetic dipole moment (MDM).

The present experimental upper limits on the branching ratios of $\mu - e$ conversion for nuclei titanium [36], gold [37] and lead [38] are

$$\text{Br}(\mu^- + \text{Ti} \rightarrow e^- + \text{Ti}) < 4.3 \times 10^{-12} \text{ (90\% C.L.) [SINDRUM II]} , \quad (26)$$

$$\text{Br}(\mu^- + \text{Au} \rightarrow e^- + \text{Au}) < 7 \times 10^{-13} \text{ (90\% C.L.) [SINDRUM II]} , \quad (27)$$

$$\text{Br}(\mu^- + \text{Pb} \rightarrow e^- + \text{Pb}) < 4.6 \times 10^{-11} \text{ (90\% C.L.) [SINDRUM II]} . \quad (28)$$

And the projected sensitivities for aluminum and titanium are [39–43]

$$\text{Br}(\mu^- + \text{Al} \rightarrow e^- + \text{Al}) < 3 \times 10^{-17} \text{ (Mu2e, COMET)} , \quad (29)$$

$$\text{Br}(\mu^- + \text{Ti} \rightarrow e^- + \text{Ti}) < 10^{-18} \text{ (Mu2e II, PRISM)} . \quad (30)$$

The current limit [44] and projected sensitivity [45] for $\text{Br}(\mu \rightarrow e\gamma)$ from MEG experiment are

$$\text{Br}(\mu \rightarrow e\gamma) \leq 4.2 \times 10^{-13} \text{ (90\% C.L.) [MEG, 2016]}, \quad (31)$$

$$\text{Br}(\mu \rightarrow e\gamma) \sim 4 \times 10^{-14} \quad [\text{Projected Sensitivity}]. \quad (32)$$

For the muon anomalous magnetic dipole moment, we have from E821 experiment [46] the 3.6σ discrepancy between the measurement and the SM prediction

$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 288(63)(49) \times 10^{-11}, \quad (33)$$

where the first errors are experimental and the second systematic. In the numerical work, we will combine the two errors in quadrature.

- The decay length (see Appendix B) of the mirror lepton is also computed. In Fig. (2), we show the contour plots of 0.15, 0.5, 1mm and 1 cm for the decay length of $e^M \rightarrow l_i + \phi_{kS}$ on the (g_{0S}, m_M) plane. We sum over all i and k and set $|g_{0S}| = |g'_{0S}| = |g_{1S}| = |g'_{1S}|$ for simplicity. In the limit of $m_M \gg m_S$, the decay length is sensitive to neither the two scenarios mentioned above nor the neutrino mass hierarchies and the CP phases.

In Fig. (3), we plot the contour of the electron EDM set to be the current ACME limit on the (g_{0S}, m_M) plane for the normal hierarchy in Scenario A (left panel) and B (right panel). We set $|g_{0S}| = |g'_{0S}| = |g_{1S}| = |g'_{1S}|$ for simplicity.

In Fig. (4), we plot the electron EDM as function of $\log_{10} |g_{0S}|$ in Scenario A and normal hierarchy for the three cases of

- (a) $\alpha = \beta = 0$ (upper left panel),
- (b) $\alpha = \beta = \pi/4$ (upper right right) and
- (c) $\alpha = \beta = 3\pi/2$ (lower panel).

Different color represents different mirror lepton mass m_M as indicated by the

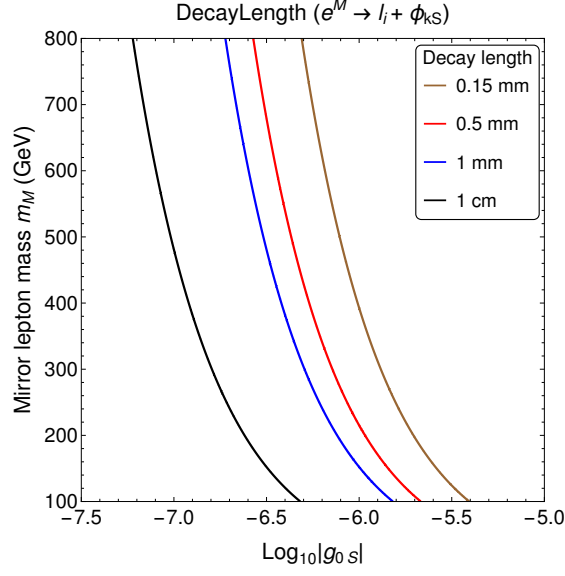


FIG. 2. Contour plot for decay length of $e^M \rightarrow l + \phi_S$ on the $(\log_{10} |g_{0S}|, m_M)$ plane with $|g_{0S}| = |g'_{0S}| = |g_{1S}| = |g'_{1S}|$.

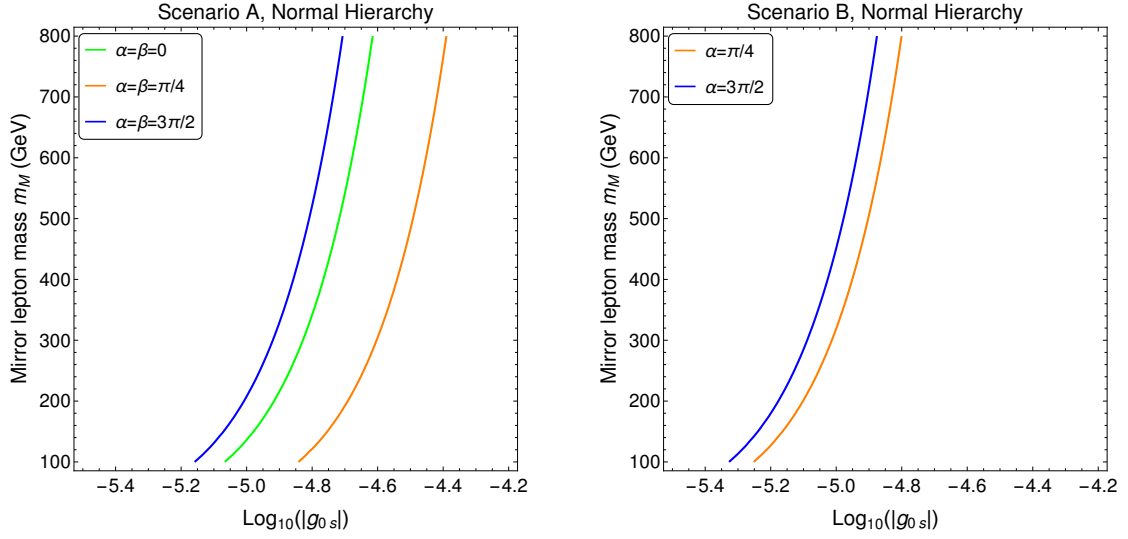


FIG. 3. Contour plot for electron EDM set to be current upper limit at ACME Collaboration on the $(\log_{10} |g_{0S}|, m_M)$ plane with $|g_{0S}| = |g'_{0S}| = |g_{1S}| = |g'_{1S}|$. Left panel (Scenario A, normal hierarchy) and Right panel (Scenario B, normal hierarchy).

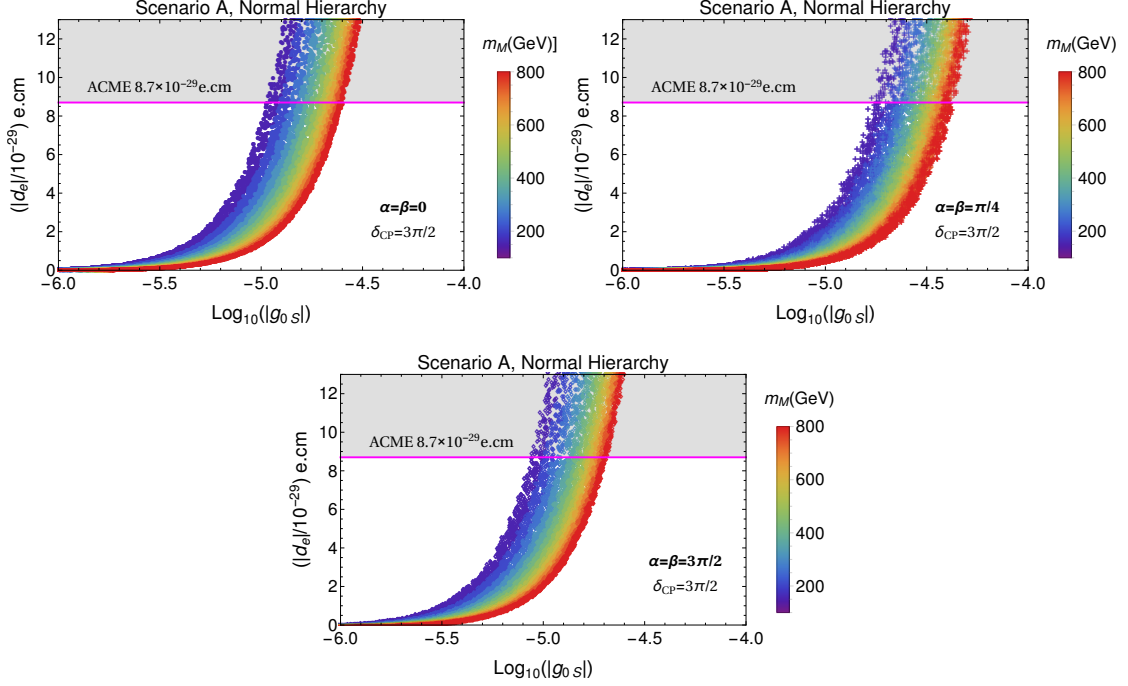


FIG. 4. Electron EDM versus $\log_{10} |g_{0S}|$ in the Scenarios A and normal hierarchy for various combination of CP phases. The pink line is current upper limit of electron EDM from ACME Collaboration [7]. The color pattern represents to various values of mirror lepton mass m_M in logarithmic scale. We set $|g_{0S}| = |g'_{0S}| = |g_{1S}| = |g'_{1S}|$.

palette at the right side of each plot. The pink line is the current limit of electron EDM from ACME. Results for inverted hierarchy are similar and will not be shown.

Fig. (5) is the same as Fig. (4) for Scenario B with $\alpha = \pi/4$ (left panel) and $3\pi/2$ (right panel). Note that for Scenario B, from (24) and (41) in Appendix C, the electron EDM is independent of β and vanishes for $\alpha = 0$.

In Fig. (6) and Fig. (7), we show the constraints on the magnitude of the couplings and their phases from the current limits and projected sensitivities of $\mu - e$ conversion, $\mu \rightarrow e\gamma$ and electron EDM from various experiments for normal hierarchy with $m_M = 200$ GeV in both Scenario A and B respectively. We set $\alpha = \beta$ for simplicity. Results for negative $\alpha(\beta)$ are symmetric to those of positive

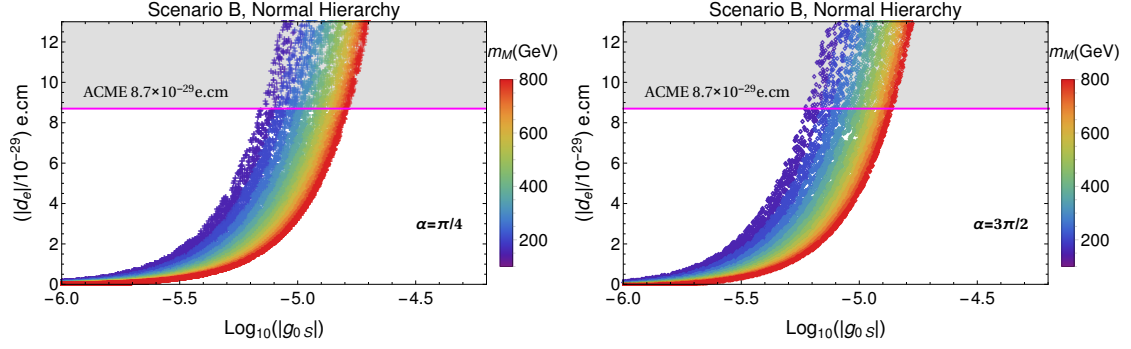


FIG. 5. Same as Fig. (4) for Scenario B with $\alpha = \pi/4$ and $3\pi/2$.

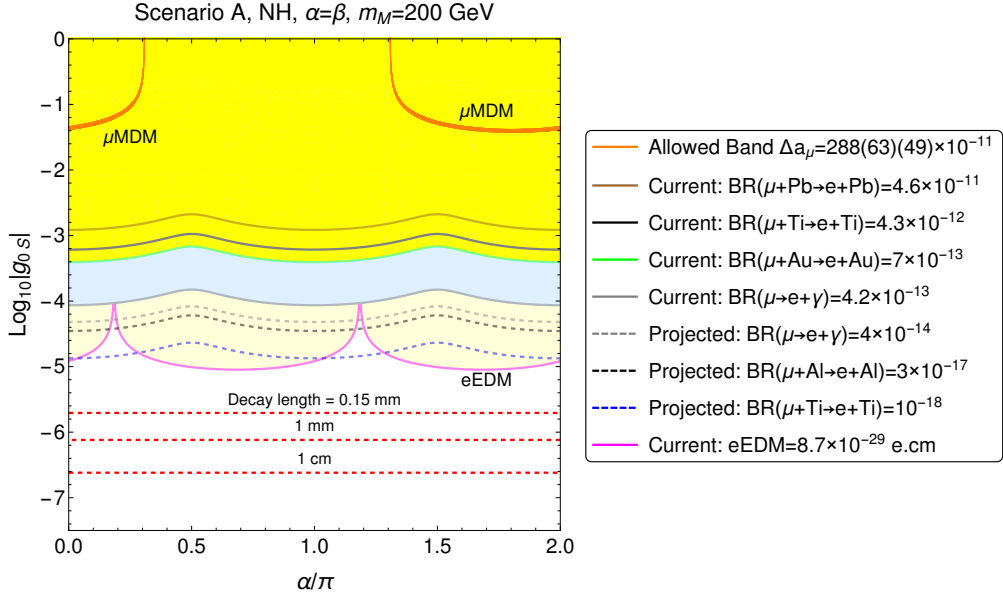


FIG. 6. Constraints from the current limits and project sensitivities of $\mu - e$ conversion, $\mu \rightarrow e\gamma$ and electron EDM on the magnitude of the couplings and their phases in normal mass hierarchy with Scenario A for $\alpha = \beta$, $\delta_{CP} = 3\pi/2$ and mirror lepton mass $m_M = 200$ GeV. The straight dashed lines are the decay length of various values for a 200 GeV mirror electron. The two orange bands are the allowed regions of the muon anomalous magnetic dipole moment.

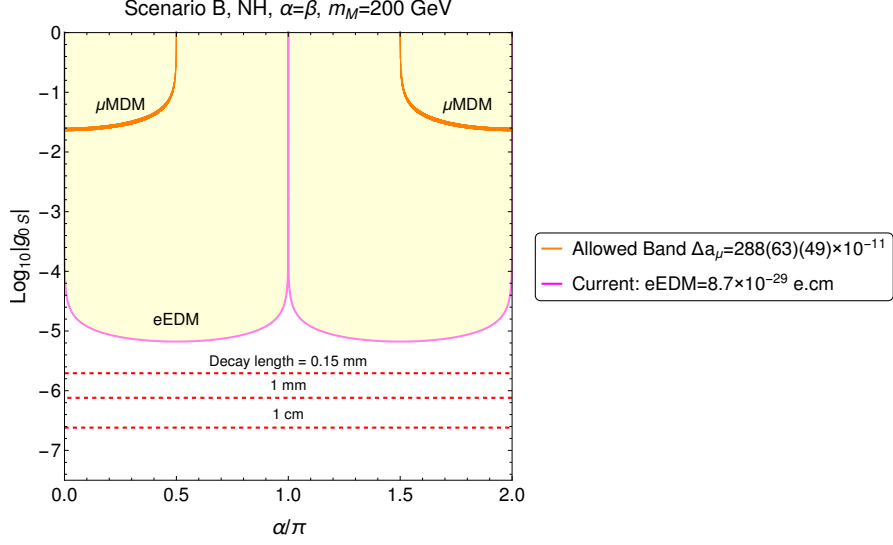


FIG. 7. The same as Fig. (6) but for Scenario B.

$\alpha(\beta)$ and will not be shown. We note that from (45) and (48) in Appendix E, both $\mu \rightarrow e\gamma$ and $\mu - e$ conversion are approximately vanishing for $m_M \gg m_S$ in Scenario B. The two thin orange bands are the 3.6σ discrepancy of the muon anomaly given in (33). However, they are not favored by the CLFV processes which constrains the couplings to be much smaller. We also note that the CP phases can enter into the muon anomaly calculation as in the case of MSSM case [21].

Since there is no significance difference for both neutrino mass hierarchies, we only show the normal hierarchy case for all plots. Results for other values of the mirror fermion mass are qualitatively the same and will be omitted too.

V. CONCLUSION

We have studied the electron EDM in the mirror fermion model with electroweak scale non-sterile right-handed neutrinos and a horizontal A_4 symmetry in the lepton and scalar sectors. Modulo the possibility of cancellation in the var-

ious CP violation phases in different scenarios such that the quantity J_e defined in Eq. (25) vanishes, current experimental limit on the electron EDM imposes the most stringent constraints on the parameter space of the model, as compared with other low energy precision observables like $\mu \rightarrow e\gamma$ and $\mu - e$ conversion in nuclei. However, projected sensitivities of $\mu \rightarrow e\gamma$ from MEG and of $\mu - e$ conversion experiments at Mu2e, Mu2e II, COMET and PRISM, can provide comparable if not more stringent constraints on the parameters of the mirror fermion model.

The region of parameter space that can “explain” the muon anomalous magnetic dipole moment in the mirror fermion model is not favored by the current limits of these CLFV processes and the electron EDM from various experiments, which suggest much smaller couplings of order 10^{-3} to 10^{-5} .

On the other hand, the parameter space that can be probed by current and near future experiments for these CLFV processes and the electron EDM is close to the region where the mirror leptons, when produced at the LHC, have the decay length of about 1 mm and hence can be prompted. The search strategies for these mirror leptons at the LHC [14] may have to include displaced vertices located at distances from 1 mm to 1 cm. It is interesting to note that SM background is expected to be small in this region and signatures for mirror leptons could be distinctive. It is also interesting to see how, in the mirror model, low energy experiments (rare processes, electron EDM) guide the direct searches at high energy (LHC) for new particles such as the mirror leptons.

For the two scenarios that we considered in this work, our results are not sensitive to the two neutrino mass hierarchies.

Besides EDM, the new CP violating Yukawa couplings studied here may have implications for leptogenesis via the asymmetry of two CP conjugate rates of mirror lepton decay $l_m^M \rightarrow l_i \phi_{kS}$ and $\bar{l}_m^M \rightarrow \bar{l}_i \phi_{kS}^*$. We would like to return to this issue in the future.

To complete the story, one might want to extend the A_4 symmetry to the quark

sector. The experimental constraints from the well established quark mixings in the CKM model must then be faced [47]. Needless to say, it is also of interest to study the EDM of neutron in the mirror model. Work on this is now in progress and will be report elsewhere.

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APPENDIX

In this Appendix, we collect some useful formulas used in this work.

(A) C_L^{ij} and C_R^{ij}

These coefficients were computed in [16] and we collect their expressions here for convenience.

$$C_L^{ij} = +\frac{e}{16\pi^2} \sum_{k=0}^3 \sum_{m=1}^3 \left\{ \frac{1}{m_{l_m^M}^2} \left[m_i \mathcal{U}_{jm}^{Rk} (\mathcal{U}_{im}^{Rk})^* + m_j \mathcal{U}_{jm}^{Lk} (\mathcal{U}_{im}^{Lk})^* \right] \mathcal{I} \left(\frac{m_{\phi_{kS}}^2}{m_{l_m^M}^2} \right) + \frac{1}{m_{l_m^M}} \mathcal{U}_{jm}^{Rk} (\mathcal{U}_{im}^{Lk})^* \mathcal{J} \left(\frac{m_{\phi_{kS}}^2}{m_{l_m^M}^2} \right) \right\} , \quad (34)$$

$$C_R^{ij} = +\frac{e}{16\pi^2} \sum_{k=0}^3 \sum_{m=1}^3 \left\{ \frac{1}{m_{l_m^M}^2} \left[m_i \mathcal{U}_{jm}^{Lk} (\mathcal{U}_{im}^{Lk})^* + m_j \mathcal{U}_{jm}^{Rk} (\mathcal{U}_{im}^{Rk})^* \right] \mathcal{I} \left(\frac{m_{\phi_{kS}}^2}{m_{l_m^M}^2} \right) + \frac{1}{m_{l_m^M}} \mathcal{U}_{jm}^{Lk} (\mathcal{U}_{im}^{Rk})^* \mathcal{J} \left(\frac{m_{\phi_{kS}}^2}{m_{l_m^M}^2} \right) \right\} , \quad (35)$$

where $m_{i,j}$, $m_{l_m^M}$ and $m_{\phi_{kS}}$ are the SM leptons, mirror leptons and scalar singlets masses respectively, with the subscripts i, j, m being the generation indices. In the calculation [16], we have assumed $m_{l_m^M} \gg m_{i,j}$ and set $m_{i,j} \rightarrow 0$ in the loop

functions $\mathcal{I}(r)$ and $\mathcal{J}(r)$, which are simply given by

$$\mathcal{I}(r) = \frac{1}{12(1-r)^4} \left[-6r^2 \log r + r(2r^2 + 3r - 6) + 1 \right] , \quad (36)$$

$$\mathcal{J}(r) = \frac{1}{2(1-r)^3} \left[-2r^2 \log r + r(3r - 4) + 1 \right] . \quad (37)$$

Note that $\mathcal{I}(0) = 1/12$ and $\mathcal{J}(0) = 1/2$.

(B) Decay Length of Mirror Leptons

For an unstable relativistic particle, its decay length l is given by $l = \beta\gamma c\tau$, where $\beta = v/c$ is its velocity, $\gamma = 1/(1-\beta^2)^{1/2}$ its dilation factor, and $\tau = (\sum \Gamma)^{-1}$ its total lifetime with Γ being its partial width.

The decay rate for $l_m^M \rightarrow l_i + \phi_{kS}$ is given by

$$\begin{aligned} \Gamma(m \rightarrow ik) &= \frac{1}{32\pi} m_{l_m^M} \left(1 - \left(\frac{m_{l_i} + m_{\phi_k}}{m_{l_m^M}} \right)^2 \right)^{1/2} \left(1 - \left(\frac{m_{l_i} - m_{\phi_k}}{m_{l_m^M}} \right)^2 \right)^{1/2} \\ &\times \left\{ \left(1 + \frac{m_{l_i}^2 - m_{\phi_k}^2}{m_{l_m^M}^2} \right) ((\mathcal{U}_{im}^{Lk})^* \mathcal{U}_{im}^{Lk} + (\mathcal{U}_{im}^{Rk})^* \mathcal{U}_{im}^{Rk}) \right. \\ &\quad \left. + \left(2 \frac{m_{l_i}}{m_{l_m^M}} \right) ((\mathcal{U}_{im}^{Lk})^* \mathcal{U}_{im}^{Rk} + (\mathcal{U}_{im}^{Rk})^* \mathcal{U}_{im}^{Lk}) \right\} . \end{aligned} \quad (38)$$

(C) The Quantity J_e of Eq. (25)

For the electron in Scenario A, we have

$$J_e = |g_{0S}| |g'_{0S}| \left(C_1 \sin(\alpha) + C_2 \sin(\delta_{\text{CP}} - \alpha) \right) + 2 |g_{1S}| |g'_{1S}| C_2 \sin(\delta_{\text{CP}}) \cos(\beta) \quad (39)$$

with

$$\begin{aligned} C_1 &= \frac{1}{\sqrt{3}} \left[c_{12}c_{13} + s_{12}(s_{23} - c_{23}) \right] , \\ C_2 &= \frac{1}{\sqrt{3}} c_{12}s_{13}(s_{23} + c_{23}) . \end{aligned} \quad (40)$$

For Scenario B, we simply have

$$J_e = |g_{0S}| |g'_{0S}| \sin(\alpha) . \quad (41)$$

Note that in Scenario B, only the magnitudes of the A_4 singlet couplings g_{0S} and g'_{0S} and their relative phase α entered in (41), the Dirac phase δ_{CP} and the A_4 -triplet couplings g_{1S} and g'_{1S} do not contribute!

(D) Muon Anomaly

For Scenario A, the muon anomaly is given by

$$\begin{aligned} \Delta a_\mu^A \approx & \frac{1}{16\pi^2} \left\{ \frac{m_\mu^2}{6m_M^2} \left[|g_{0S}|^2 + |g'_{0S}|^2 + 2(|g_{1S}|^2 + |g'_{1S}|^2) \right] \right. \\ & + \frac{m_\mu}{4m_M} \left[|g_{0S}| |g'_{0S}| \left(C_3 \cos(\alpha) + C_4 \sin(\alpha) + C_5 \sin(\delta_{\text{CP}} - \alpha) + C_6 \cos(\delta_{\text{CP}} - \alpha) \right) \right. \\ & \left. \left. + 2 |g_{1S}| |g'_{1S}| \cos(\beta) \left(C_3 + C_5 \sin(\delta_{\text{CP}}) + C_6 \cos(\delta_{\text{CP}}) \right) \right] \right\} , \end{aligned} \quad (42)$$

where

$$\begin{aligned} C_3 &= \frac{1}{\sqrt{3}} \left(-c_{12}c_{23} + c_{12}s_{23} + 2s_{12}c_{13} \right) , \\ C_4 &= -c_{12}(s_{23} + c_{23}) , \\ C_5 &= s_{12}s_{13}(c_{23} - s_{23}) , \\ C_6 &= \frac{1}{\sqrt{3}} s_{12}s_{13}(c_{23} + s_{23}) . \end{aligned} \quad (43)$$

For Scenario B, we have

$$\begin{aligned} \Delta a_\mu^B \approx & \frac{1}{16\pi^2} \left\{ \frac{m_\mu^2}{6m_M^2} \left[|g_{0S}|^2 + |g'_{0S}|^2 + 2(|g_{1S}|^2 + |g'_{1S}|^2) \right] \right. \\ & \left. + \frac{m_\mu}{2m_M} \left[|g_{0S}| |g'_{0S}| \cos(\alpha) + 2 |g_{1S}| |g'_{1S}| \cos(\beta) \right] \right\} . \end{aligned} \quad (44)$$

Exact analytical formula of the anomalous MDM for lepton l_i can be found in [16].

(E) $\mu - e$ Conversion and Radiative Decay $\mu \rightarrow e\gamma$

The branching ratios for the $\mu - e$ conversion rate and $\mu \rightarrow e\gamma$ are related as [18]

$$\text{Br}(\mu N \rightarrow eN) = \frac{\Gamma_{\text{conv}}^{\gamma^*}}{\Gamma_{\text{capt}}} \approx \pi D^2 \frac{\Gamma_{\mu}}{\Gamma_{\text{capt}}} \text{Br}(\mu \rightarrow e\gamma) \quad (45)$$

where Γ_{capt} and D are the capture rate and overlap integral of the nuclei N respectively. Their values for different nuclei are listed in Table IV for convenience. Γ_{μ} is the total width of the muon. Detailed analytical expressions for $\mu - e$ conversion in nuclei can be found in [18].

For Scenario A, we have

$$\begin{aligned} \Gamma^A(\mu \rightarrow e\gamma) \approx & \frac{1}{16\pi} m_{\mu}^3 \left(1 - \frac{m_e^2}{m_{\mu}^2}\right)^3 \left(\frac{e}{32\pi^2 m_M}\right)^2 \left(C_7 + C_8 \cos(\delta_{\text{CP}}) + C_9 \sin(\delta_{\text{CP}})\right) \\ & \times \left[|g_{0S}|^2 |g'_{0S}|^2 + 2 |g_{1S}|^2 |g'_{1S}|^2 (1 + \cos(2\beta)) \right. \\ & \left. + 4 |g_{0S}| |g'_{0S}| |g_{1S}| |g'_{1S}| \cos(\alpha) \cos(\beta) \right], \quad (46) \end{aligned}$$

where

$$\begin{aligned} C_7 &= \frac{1}{3} \left(2 - c_{12}^2 c_{23} s_{23} (s_{13}^2 + 2) + 3 c_{12} c_{13} s_{12} (c_{23} - s_{23}) + s_{12}^2 c_{23} s_{23} (2 s_{13}^2 + 1) \right), \\ C_8 &= \frac{1}{3} \left(s_{13} (c_{23} + s_{23}) (c_{12}^2 c_{13} + 3 c_{12} s_{12} (s_{23} - c_{23}) - 2 c_{13} s_{12}^2) \right), \\ C_9 &= \frac{1}{\sqrt{3}} c_{12} s_{13} \left(c_{12} c_{13} (c_{23} - s_{23}) + s_{12} \right). \quad (47) \end{aligned}$$

| Nuclei | $\Gamma_{\text{capt}} (10^6 \text{ s}^{-1})$ | D |
|------------------------|--|--------|
| $^{48}_{22}\text{Ti}$ | 2.59 | 0.0864 |
| $^{197}_{79}\text{Au}$ | 13.07 | 0.189 |
| $^{208}_{82}\text{Pb}$ | 13.45 | 0.161 |

TABLE IV. SM values of the capture rates (in unit of 10^6 s^{-1} [48]) and the dimensionless overlap integrals (evaluated under the assumption that the proton and neutron distributions within each nuclei are the same [49]) for titanium, gold and lead.

We note that the CP violation phases enter into the decay rate of $\mu \rightarrow e\gamma$ in Eq. (46).

For Scenario B, we have

$$\Gamma^B(\mu \rightarrow e\gamma) \approx 0. \quad (48)$$

Perhaps this null result needs some explanations. Note that the amplitude for $l_i \rightarrow l_j + \gamma$ ($i \neq j$) is proportional to the off-diagonal elements of $C_{L,R}^{ij}$. However, under the set up of the parameters space discussed in Sect. IV, $C_{L,R}^{ij}$ in Scenario B ($C_{L,R}^{B,ij}$) are diagonal matrices in i, j . For example,

$$\begin{aligned} C_L^{B,ij} &\approx \sum_{k=0}^3 \sum_{m=1}^3 \left\{ a \mathcal{U}_{jm}^{Rk} (\mathcal{U}_{im}^{Rk})^* + b \mathcal{U}_{jm}^{Lk} (\mathcal{U}_{im}^{Lk})^* + c \mathcal{U}_{jm}^{Rk} (\mathcal{U}_{im}^{Lk})^* \right\} \\ &= \sum_{k=0}^3 \left\{ U_{\text{PMNS}}^\dagger \left[a M'^k (M'^k)^\dagger + b M^k (M^k)^\dagger + c M'^k (M^k)^\dagger \right] U_{\text{PMNS}} \right\}_{ji} \end{aligned} \quad (49)$$

where a, b, c are some constants related to the masses. Furthermore one can easily check $\sum_{k=0}^3 M'^k (M'^k)^\dagger$, $\sum_{k=0}^3 M^k (M^k)^\dagger$ and $\sum_{k=0}^3 M'^k (M^k)^\dagger$ are diagonal matrices. Since U_{PMNS} is unitary, this implies $C_L^{B,ij}$ is diagonal. Similarly, $C_R^{B,ij}$ is also diagonal. Thus $\Gamma^B(\mu \rightarrow e\gamma) \approx 0$, and there is no $\mu - e$ conversion as well in Scenario B, according to Eq. (45).

Exact formulas for the rate of $l_i \rightarrow l_j + \gamma$ can be found in [16].

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